

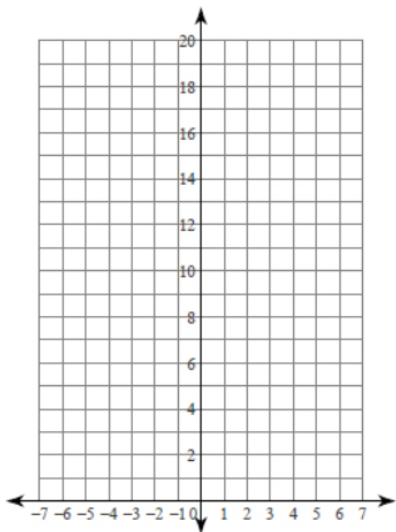
EXPONENTIAL FUNCTIONS

Day 1
Front

$$Y=AB^X$$

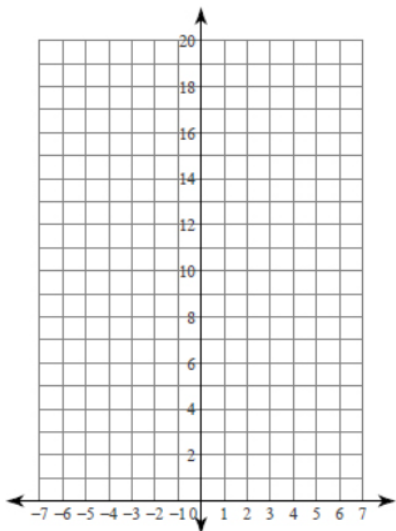
$$y = 4(2)^x$$

x	y



$$y = 4\left(\frac{1}{2}\right)^x$$

x	y



EXPONENTIAL GROWTH AND DECAY

Day 2
Front

$$Y=A(1\pm R)^X$$

A=START

R=PERCENT CHANGE

X=HOW OFTEN CHANGE OCCURS

Y=RESULT OF CHANGE OVER TIME

1. The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.2%. If the world population continued to grow this fast, how many people would be on Earth now?

2. A computer is purchased for \$2000 but loses 20% of its value each year. How much will it be worth in 4 years?

3. Movie tickets now average \$9.75 a ticket, but are increasing in cost by 15% per year. How much will they cost in 5 years?

4. A Honda Accord depreciates at 18% per year. Six years ago it was purchased for \$21,000. What is it worth now?

A=START
B=CHANGE
X=HOW OFTEN CHANGE OCCURS
Y=RESULT OF CHANGE OVER TIME

1. March Madness is an example of exponential decay. At each round of the tournament, only the winning teams stay, so the number of teams playing at each round is half of the number of teams playing in the previous round. If 64 teams are a part of the official bracket at the start, how many teams are left after 5 rounds of play?

2. Bacteria have the ability to multiply at an alarming rate, where each bacteria splits into two new cells, doubling the number of bacteria present. If there are ten bacteria on your desk, and they double every hour, how many bacteria will be present tomorrow (desk uncleaned)?

3. The zombie apocalypse has begun! Every month, Atlanta survivors take a census to keep a record of the new human population. There are currently 450,000 people living in Atlanta. One month after the zombie outbreak, there are only approximately 90,000 people. One more month later, there are only 18,000 people. How many people do you think would survive the zombie apocalypse in Atlanta for a year?

4. Phosphorus-32 is used to study a plant's use of fertilizer. It has a half-life of 14 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

EXPONENTIAL MULTIPLE CHOICE PRACTICE

1. A population of bacteria can be modeled by the function $f(t) = 1000(0.98)^t$, where t represents the time since the population started decaying, and $f(t)$ represents the population of the remaining bacteria at time t . What is the rate of decay for this population?

- a. 98%
- b. 2%
- c. 0.98%
- d. 0.02%

2. The equation $V(t) = 12,000(0.75)^t$ represents the value of a motorcycle t years after it was purchased. Which statement is true?

- a. The motorcycle cost \$9000 when purchased.
- b. The motorcycle cost \$12,000 when purchased.
- c. The motorcycle's value is decreasing at a rate of 75% each year.
- d. The motorcycle's value is decreasing at a rate of 0.25% each year.

3. Marc bought a new laptop for \$1250. He kept track of the value of the laptop over the next three years, as shown in the table below. Which function can be used to determine the value of the laptop for x years after the purchase?

- a. $f(x) = 1000(1.2)^x$
- b. $f(x) = 1250(1.2)^x$
- c. $f(x) = 1000(0.8)^x$
- d. $f(x) = 1250(0.8)^x$

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

4. A certain population of bacteria has an average growth rate of 2%. The formula for the growth of the bacteria's population is _____ . If you begin with 200 bacteria, about how many bacteria will there be after 100 hours?

- a. 7
- b. 272
- c. 1,449
- d. 20,000

SEQUENCES

A sequence is a list of numbers or objects, called _____, in a certain order.

For **arithmetic sequences**, the difference between any two _____ terms is always the same. This number is called a _____, the number being _____ to any term to achieve the next term.

For **geometric sequences**, the _____ of consecutive terms is always the same. This number is called the _____, the number being _____ to any term to achieve the next term.

Identify the following sequences as arithmetic or geometric. Then name the common difference or common ratio.

- | | |
|--------------------------|------------------------------|
| 1. 2, 6, 10, 14, ... | 2. 2, 6, 18, 54, ... |
| 3. 56, 84, 126, 189, ... | 4. 56, 26, -4, -34, ... |
| 5. 25, 75, 125, 175, ... | 6. 25, 75, 225, 675, ... |
| 7. 0.1, 1, 10, 100, ... | 8. 0.1, 0.15, 0.2, 0.25, ... |

RECURSIVE FORMULAS

A recursive formula is a formula that says how to determine the **next** term based on the **previous** term.

Arithmetic Sequence Recursive Formula $\begin{cases} a_1 = \\ a_n = a_{n-1} + d \end{cases}$

Geometric Sequence Recursive Formula $\begin{cases} a_1 = \\ a_n = r \cdot a_{n-1} \end{cases}$

Match the following recursive formulas with their sequences.

- | | |
|-----------------------------|--------------------------|
| 1. 5, 15, 25, 35, ... | a. $a_n = a_{n-1} - 2.5$ |
| 2. 8, -20, 50, -125, ... | b. $a_n = a_{n-1} + 2$ |
| 3. 5, 15, 45, 135, ... | c. $a_n = a_{n-1} + 5$ |
| 4. 20, 17.5, 15, 12.5, ... | d. $a_n = a_{n-1} + 10$ |
| 5. -8, -3, 2, 7, ... | e. $a_n = 3a_{n-1}$ |
| 6. 1000, 500, 250, 125, ... | f. $a_n = 0.5a_{n-1}$ |
| 7. -99, -97, -95, -93, ... | g. $a_n = 5a_{n-1}$ |
| 8. 2, 10, 50, 250, ... | h. $a_n = -2.5a_{n-1}$ |

FORMULAS

An explicit formula is a formula that allows you to find **any** term.

Arithmetic Sequence Explicit Formula $a_n = a_1 + d(n - 1)$

Geometric Sequence Explicit Formula $a_n = a_1 \cdot r^{n-1}$

Example 1: Find the 21st term of the sequence 32, 26, 20, 14, 8, ...

Example 2: Find the 11th term of the sequence 1024, 512, 256, ...

Find the given term of each of the following sequences.

1. Given the sequence 25, 40, 55, 70, ... what is the 24th term?
2. Given the sequence 0.01, 0.2, 4, 80, ... what is the 9th term?
3. Given the sequence 88, 81, 74, 67, ... what is the 18th term?
4. Given the sequence 384, 96, 24, 6, ... what is the 7th term?

Write a recursive formula for the following sequences.

1. 3, -6, 12, -24, ...
2. 3, -9, -21, -33, ...

Use the given formulas to generate the first four terms of the corresponding sequences.

3.
$$\begin{cases} a_1 = 54 \\ a_n = \frac{1}{3}a_{n-1} \end{cases}$$

4.
$$\begin{cases} a_1 = 10 \\ a_n = a_{n-1} + 3 \end{cases}$$

5.
$$\begin{cases} a_1 = 10 \\ a_n = 3a_{n-1} \end{cases}$$

Sequences Multiple Choice Practice

1. The formula of the n th term of the sequence $3, -6, 12, -24, 48, \dots$ is

a. $a_n = -2(3)^n$

c. $a_n = -2(3)^{n-1}$

b. $a_n = 3(-2)^n$

d. $a_n = 3(-2)^{n-1}$

2. What is a formula for the n th term of sequence B shown below?

$$B = 10, 12, 14, 16, \dots$$

a. $b_n = 8 + 2n$

c. $b_n = 10(2)^n$

b. $b_n = 10 + 2n$

d. $b_n = 10(2)^{n-1}$

3. A sequence has the following terms: $a_1 = 4, a_2 = 10, a_3 = 25, a_4 = 62.5$. Which formula represents the n th term in the sequence?

a. $a_n = 4 + 2.5n$

c. $a_n = 4(2.5)^n$

b. $a_n = 4 + 2.5(n - 1)$

d. $a_n = 4(2.5)^{n-1}$

4. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1 , which is an equation for the n th term of this sequence?

a. $a_n = 8n + 10$

c. $a_n = 16n + 10$

b. $a_n = 8n - 14$

d. $a_n = 16n - 38$

5. At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the n th term of this sequence is $a_n = 25000 + 1000(n - 1)$. Which rule best represents the equivalent recursive formula?

a. $a_1 = 25000; a_n = 1000a_{n-1}$

c. $a_1 = 25000; a_n = a_{n-1} + 1000$

b. $a_1 = 1000; a_n = 25000a_{n+1}$

d. $a_1 = 25000; a_n = a_{n+1} + 1000$

6. The formula $\begin{cases} a_1 = 3000 \\ a_n = 0.80a_{n-1} \end{cases}$ can be used to model which scenario?

- a. The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
- b. A bank account starts with a deposit of \$3000, and each year it grows by 80%.
- c. The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
- d. The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

7. Which function represents this sequence?

n	1	2	3	4	5	...
a_n	-1	1	3	5	7	...

- a. $a_n = a_{n-1} + 1$
- b. $a_n = a_{n-1} + 2$
- c. $a_n = 2a_{n-1}$
- d. $a_n = 2a_{n-1} - 3$

8. A theater has more seats in the back rows than it has in the front rows. At a particular theater each row has two more seats than the row in front of it. Which formulas model this situation if the front row has twenty seats?

- a. $a_n = a_{n-1} + 2$ and $a_n = 2n + 20$
- b. $a_n = a_{n-1} + 2$ and $a_n = 2n + 18$
- c. $a_n = 2a_{n-1}$ and $a_n = 2n + 20$
- d. $a_n = 2a_{n-1}$ and $a_n = 2n + 18$

9. Select TWO of the following statements that are TRUE based on the following pictorial sequence.



- a. $a_n = 2a_{n-1}$
- b. $a_n = a_{n-1} + 2$
- c. $a_n = a_{n-1} - 2$
- d. $a_n = 3a_{n-1}$
- e. $a_n = 2n + 1$
- f. $a_n = 2n + 3$